Generalized and exact solutions for oblique shock waves of real gases with application to real air

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Calculation of the oblique shock wave of real gases is a difficult and time consuming problem because it involves numerical solution of a set of 10 equations, two of which (i.e., the equation of state and enthalpy function)—if available—are of a very complicated algebraic form. The present work presents a generalized method for calculating oblique shock waves of real gases, based on the Redlich–Kwong equation of state. Also described is an exact method applicable when the exact equation of state and enthalpy function of a real gas are available. Application of the generalized and the exact methods in the case of real air showed that the former is very accurate and at least twenty times faster than the latter. An additional contribution of the study is the derivation of real gas oblique shock wave equations, which are of the same algebraic form as the well known ideal gas normal shock wave relations.

Keywords: oblique shock; real gases; air

Introduction

Oblique shock waves, i.e., pressure discontinuities inclined to the direction of the oncoming compressible flow, may occur in almost all supersonic flow patterns of practical significance. With reference to Figure 1, C_1 and C_2 denote the stream velocities entering and leaving the oblique shock wave, respectively, δ is the flow deflection angle and σ, σ-δ are the angles between the shock wave and the vectors C_1, C_2, respectively. The components of the velocities C_1 and C_2 resolved in the tangential and normal directions to the oblique shock wave are denoted by C_{1t}, C_{2t}, C_{1n}, and C_{2n}, as shown in Figure 1. If p, v, T, and h stand for the fluid pressure, specific volume, temperature and specific enthalpy, respectively, and subscripts 1 and 2 denote quantities upstream and downstream the shock wave, the following equations are valid:

Equation of state:

$$p = p(v, T) \tag{1}$$

Enthalpy function:

$$h = h(v, T) \tag{2}$$

Continuity of mass:

$$v_2 C_{1n} = v_1 C_{2n} \tag{3}$$

Conservation of momentum in the normal and tangential directions to the oblique shock wave:

$$p_1 + C_{1n}^2/v_1 = p_2 + C_{2n}^2/v_2 \tag{4}$$

$$C_{1t} = C_{2t} \tag{5}$$

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Conservation of energy:

$$h_1 + C_{1n}^2/2 = h_2 + C_{2n}^2/2 \tag{6}$$

Velocity components:

$$C_{1t} = C_1 \cos\sigma \tag{7}$$

$$C_{1n} = C_1 \sin\sigma \tag{8}$$

$$C_{2t} = C_2 \cos(\sigma - \delta) \tag{9}$$

$$C_{2n} = C_2 \sin(\sigma - \delta) \tag{10}$$

Equations (1) to (10) comprise a set of 10 equations for the 10 unknowns p_2, v_2, T_2, h_2, C_2, C_{2n}, C_{2t}, C_{1n}, C_{1t} and σ in terms of the known upstream conditions p_1, v_1, T_1, h_1, C_1 and the known flow deflection angle δ. In the case of a real gas, solution of the above equation set is difficult and can be obtained only numerically, mainly because of the complexity of Equations 1 and 2. Thus, shock wave tables are available for only a few real gases, usually restricted to the limiting case of the normal shock wave (i.e., for $\sigma = 90°$, $\delta = 0$), as for example for real air.[1] The lack of oblique shock wave tables for most

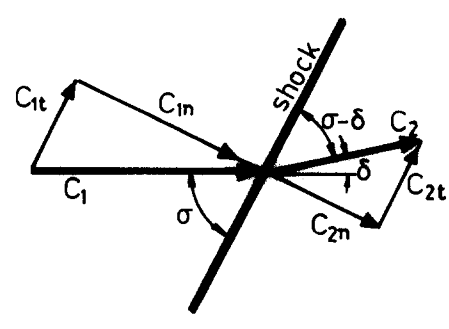

Figure 1 Stream velocities C_1 and C_2 entering and leaving the oblique shock wave, respectively

real gases is due either to the difficulty of solving the 10-equation set or to the lack of Equations 1 and 2.

Here, a generalized method for calculating the oblique shock wave of real gases is presented. The method is based on the Redlich–Kwong equation of state[2] and is applicable to real gases for which the equation of state 1 and the enthalpy function 2 are not available. The Redlich–Kwong equation has been selected among other generalized equations of state (i.e., the Lee–Kesler,[3] the Redlich–Kwong–Soave,[4] the Pitzer,[5] etc. correlations) because of its simplicity and also because it gives explicit functions facilitating the calculation of the oblique shock wave.

In order to evaluate the above generalized method with respect to accuracy and economy, a numerical algorithm has been developed for solving the set of Equations 1 to 10 in the case when the exact equation of state 1 and the exact enthalpy function 2 are available. Application of this algorithm in the case of real air showed that the generalized method is both accurate and reduces drastically computation time (by at least 95%). Therefore the generalized method is recommended even if the exact forms of Equations 1 and 2 are available.

Apart for the method presented, an additional contribution of the present study is the derivation of real gas oblique shock wave equations, which are of the same form as the well known ideal gas normal shock wave equations,[6,7], as shown in Table 1 and explained in the following section.

Real gas oblique shock wave equations

The isentropic expansion of a real gas may be described very accurately by the following empirical relations,[8] which are of

Table 1 Comparison of the derived real gas/shock wave equations with the corresponding ideal gas normal shock wave equations

Ideal gas normal shock wave [6,7]		Real gas oblique shock wave	
$\frac{p_2}{p_1}=\frac{1+kM_1^2}{1+kM_2^2}$	(T.1)	$\frac{p_2}{p_1}=\frac{1+k_{pv1}M_1^2\sin^2\sigma}{1+k_{pv2}M_2^2\sin^2(\sigma-\delta)}$	(T.5)
$\frac{v_2}{v_1}=\left[\frac{1+\frac{k-1}{2}M_1^2}{1+\frac{k-1}{2}M_2^2}\right]\left[\frac{1+kM_2^2}{1+kM_1^2}\right]$	(T.2)	$\frac{v_2}{v_1}=\left[\frac{1+k_{pv1}M_1^2\sin^2\sigma}{1+k_{pv2}M_2^2\sin^2(\sigma-\delta)}\right]\left[\frac{k_{pv2}M_2^2\sin^2(\sigma-\delta)}{k_{pv1}M_1^2\sin^2\sigma}\right]$	(T.6)
$\frac{T_2}{T_1}=\frac{1+\frac{k-1}{2}M_1^2}{1+\frac{k-1}{2}M_2^2}$	(T.3)	$\frac{T_2}{T_1}=\left[\frac{1+k_{pv1}M_1^2\sin^2\sigma}{1+k_{pv2}M_2^2\sin^2(\sigma-\delta)}\right]^2\left[\frac{M_2\sin(\sigma-\delta)}{M_1\sin\sigma}\right]^2\left[\frac{Z_1k_{pv2}}{Z_2k_{pv1}}\right]$	(T.7)
$\frac{M_1\left[1+\frac{k-1}{2}M_1^2\right]^{1/2}}{1+kM_1^2}=\frac{M_2\left[1+\frac{k-1}{2}M_2^2\right]^{1/2}}{1+kM_2^2}$	(T.4)	$\left[\frac{M_1[1+nM_1^2]^{-d_1}\sin\sigma}{1+k_{pv1}M_1^2\sin^2\sigma}\right]\left[\frac{k_{pv1}}{m_1C_1^*}\right]=\left[\frac{M_2[1+nM_2^2]^{-d_2}\sin(\sigma-\delta)}{1+k_{pv2}M_2^2\sin^2(\sigma-\delta)}\right]\left[\frac{k_{pv2}}{m_2C_2^*}\right]$	(T.8)

Notation

A, B, C	Coefficients of the c_p' polynomial
A_{ij}, B_i	Constants in Equations 42 and 44
a, b	Parameters in the Redlich-Kwong equation
a_p, a_T, a_v	
g, d, e, r	Exponents and coefficients in Equations 11 to
l, m, n	17
C	Velocity
c_p	Constant pressure heat capacity
c_p'	Constant pressure heat capacity in the ideal gas state
c_v	Constant volume heat capacity
F	Area
h	Specific enthalpy
J	Impulse function
k	Ideal gas isentropic exponent, c_p/c_v
k_{pv}	Real gas isentropic exponent corresponding to the pair of variables p, v
M	Mach number
M_{ij}, N_{ij}	Constants in Equations 43 and 44
O_{ij}, Q_{ij}, q_i	Constants in Equations 45 and 43
p	Pressure
R	Gas constant
s	Specific entropy
T	Temperature
v	Specific volume
Z	Compressibility factor

Greek symbols

α	Sound velocity
δ	Flow deflection angle
Δv	Volume increment
ρ	Density
σ	Angle between shock wave and velocity C_1

Subscripts and superscripts

c	Denotes values at the critical point
n	Denotes direction normal to the shock wave
ref	Denotes a reference value
t	Denotes direction tangential to the shock wave
0	Denotes stagnation conditions
1, 2	Denote values upstream and downstream the shock wave, respectively
*	Denotes critical conditions ($M=1$)

the same algebraic form as the corresponding ideal gas equations, but with different coefficients and exponents, i.e.,

$$p/p_0=(1+nM^2)^{a_p} \tag{11}$$

$$T/T_0=(1+nM^2)^{a_T} \tag{12}$$

$$v_0/v=(1+nM^2)^{a_v} \tag{13}$$

$$F^*/F=mM(1+nM^2)^g \tag{14}$$

$$C/2C^*=mM(1+nM^2)^d \tag{15}$$

$$C^2/2vp_0=mM(1+nM^2)^e \tag{16}$$

$$J^*/J=mM(1+nM^2)^r/(1+lM^2) \tag{17}$$

where M is the Mach number; subscripts 0 and * refer to stagnation conditions and critical values ($M=1$), respectively; and F and J are the cross-sectional flow area and the impulse function, respectively. The values of coefficients l, m, n and exponents a_p, a_T, a_v, g, d, e, and r for various real gases, including steam, air, ammonia and refrigerants R12 and R22, may be found in Ref. 8.

Calculation of Mach number, M, requires knowledge of the real gas sound velocity, α, which is calculated as[9]

$$\alpha=\left(\frac{\partial p}{\partial \rho}\right)_s^{0.5}=\left[-v^2\left(\frac{\partial p}{\partial v}\right)_s\right]^{0.5}=\left[-v^2\left(\frac{c_p}{c_v}\right)\left(\frac{\partial p}{\partial v}\right)_T\right]^{0.5}$$
$$=(k_{pv}pv)^{0.5}=(Zk_{pv}RT)^{0.5} \tag{18}$$

where $Z=pv/RT$ is the compressibility factor, and k_{pv} is a real gas isentropic exponent introduced in previous publications[10–12] and given by

$$k_{pv}=-\frac{v}{p}\frac{c_p}{c_v}\left(\frac{\partial p}{\partial v}\right)_T \tag{19}$$

By use of the sound velocity expression 18, the normal velocity components C_{1n} and C_{2n} become

$$C_{1n}=C_1\sin\sigma=M_1\alpha_1\sin\sigma=M_1(k_{pv1}p_1v_1)^{0.5}\sin\sigma \tag{20}$$

$$C_{2n}=C_2\sin(\sigma-\delta)=M_2\alpha_2\sin(\sigma-\delta)=M_2(k_{pv2}p_2v_2)^{0.5}\sin(\sigma-\delta) \tag{21}$$

and substitution of C_{1n} and C_{2n} from the above equations into the continuity Equation 3, yields after rearrangement

$$\frac{v_2}{v_1}=\frac{k_{pv2}}{k_{pv1}}\frac{p_2}{p_1}\left[\frac{M_2\sin(\sigma-\delta)}{M_1\sin\sigma}\right]^2 \tag{22}$$

Momentum Equation 4, combined with continuity Equation 3 and Equation 8, gives

$$\frac{p_2}{p_1}=1+\frac{C_1^2\sin^2\sigma}{p_1v_1}\left(1-\frac{v_2}{v_1}\right) \tag{23}$$

Substitution of $C_1=M_1\alpha_1=M_1(k_{pv1}p_1v_1)^{0.5}$ and v_2/v_1 from Equation 22 into Equation 23 yields, after rearrangement, the real gas oblique shock wave equation (T.5), given in Table 1. Comparison of this equation with the corresponding ideal gas normal shock wave equation (T.1), given in the same table, shows that both equations have the same general form. They differ in that, in the case of the real gas, two different isentropic exponents k_{pv1} and k_{pv2} appear, instead of the constant ideal gas isentropic exponent k.

With reference to Table 1, the real gas oblique shock wave relation T.6 corresponding to the ideal gas normal shock wave equation, T.2, may be derived by substitution of p_2/p_1 from Equation T.5 into Equation 22. Similarly, the real gas equation, T.7, which corresponds to the ideal gas Equation T.3 is derived by substituting p_2/p_1 and v_2/v_1 from Equations T.5 and T.6, respectively, into relation $T_2/T_1=(Z_1/Z_2)(p_2/p_1)(v_2/v_1)$. Finally, the real gas equation, T.8 corresponding to the ideal gas relation T.4 is found by substitution of v_2/v_1 and C_2/C_1 from Equations T.6 and 15, respectively, into the continuity equation $v_2/v_1=C_{2n}/C_{1n}=(C_2/C_1)[\sin(\sigma-\delta)/\sin\sigma]$.

Real gas oblique shock wave by use of the Redlich–Kwong equation

Solution procedure

The Redlich–Kwong equation of state[2] has been successfully employed in Ref. 13 for calculating the one-dimensional isentropic flow of real gases. It may be written as

$$p=\frac{RT}{v-b}-\frac{a}{T^{0.5}}\frac{1}{v(v+b)} \tag{24}$$

where R is the gas constant, and parameters a and b are calculated in terms of pressure p_c and temperature T_c at the critical point of the gas under examination; i.e.,

$$a=0.4278R^2T_c^{2.5}/p_c \tag{25}$$

$$b=0.0867RT_c/p_c \tag{26}$$

The enthalpy function h and the sound velocity expression, α, based on the Redlich–Kwong equation, 24, may be expressed as

$$h=AT+\frac{B}{2}T^2+\frac{C}{3}T^3-h_{\text{ref}}+\frac{3a}{2bT^{0.5}}\ln\left[\frac{v}{v+b}\right]-RT+pv \tag{27}$$

$$\alpha=v\left[\frac{RT}{(v-b)^2}-\frac{a(2v+b)}{T^{0.5}v^2(v+b)^2}+\frac{T\left[\dfrac{R}{v-b}+\dfrac{a}{2T^{1.5}v(v+b)}\right]^2}{A-R+BT+CT^2-\dfrac{3a}{4bT^{1.5}}\ln\left[\dfrac{v}{v+b}\right]}\right]^{0.5} \tag{28}$$

where h_{ref} is a reference enthalpy and A, B, C are the coefficients of the polynomial giving the constant pressure heat capacity, c_p', in the ideal gas state, $c_p'=A+BT+CT^2$. Values of A, B, C for various real gases may be found in Ref. 14.

The conservaton of momentum equation, 4, may be written by use of continuity Equation 3 and Equation 8 as

$$p_2-p_1=\left[\frac{C_1\sin\sigma}{v_1}\right]^2(v_1-v_2) \tag{29}$$

Substitution of p_2 from the Redlich–Kwong equation, 24, into Equation 29 gives after rearrangement:

$$(T_2^{0.5})^3+\left[\frac{b-v_2}{R}\left[p_1+\frac{C_1^2\sin^2\sigma}{v_1^2}(v_1-v_2)\right]\right]T_2^{0.5}+\left[\frac{a}{R}\frac{b-v_2}{v_2(v_2+b)}\right]=0 \tag{30}$$

which is a cubic equation with respect to $T_2^{0.5}$ and admits of the following solution:

$$T_2^{0.5}=2Q^{0.5}\cos\frac{\theta}{3} \tag{31}$$

where

$$Q=\frac{v_2-b}{3R}\left[p_1+\frac{C_1^2\sin^2\sigma}{v_1^2}(v_1-v_2)\right] \tag{32}$$

$$\cos\theta=\frac{a(b-v_2)}{2Rv_2(v_2+b)Q^{1.5}} \tag{33}$$

For the calculation of the unknown v_2 contained in Equation 31, conservation of energy Equation 6 may be employed, which by use of continuity Equation 3 and Equation 8 becomes

$$h_2-h_1=\frac{C_1^2\sin^2\sigma}{2}\left[1-\frac{v_2^2}{v_1^2}\right] \tag{34}$$

Substitution of h_2 and h_1 from the enthalpy function, Equation 27, into Equation 34, and also substitution of p_2 from the Redlich–Kwong equation 24 yields

$$\begin{aligned}&(A-R)(T_2-T_1)+\frac{B}{2}(T_2^2-T_1^2)+\frac{C}{3}(T_2^3-T_1^3)+\frac{Rv_2T_2}{v_2-b}\\&-\frac{a}{T_2^{0.5}(v_2+b)}-p_1v_1\\&+\frac{3a}{2b}\left[T_2^{-0.5}\ln\left[\frac{v_2}{v_2+b}\right]-T_1^{-0.5}\ln\left[\frac{v_1}{v_1+b}\right]\right]\\&=\frac{C_1^2\sin^2\sigma}{2}\left[1-\frac{v_2^2}{v_1^2}\right]\end{aligned} \tag{35}$$

Finally, substitution of T_2 from Equation 31 into Equation 35, yields an equation containing only unknowns v_2 and σ. Therefore, if a value is given to the angle σ, then v_2 may be calculated by solving numerically the equation. Although the flow deflection angle δ is usually known and σ is computable, in the present procedure σ is taken as known and δ is calculated. Obviously, this interchange of variables, is made to facilitate the numerical procedure and does not destroy generality. Thus, the procedure for calculating the oblique shock wave of real gases, may be summarized as follows:

(a) Substitute T_2 from Equation 31 into Equation 35, give a value to σ ($0°<\sigma\leqslant 90°$) and calculate v_2 by solving numerically the resulting equation.
(b) Calculate T_2 from Equation 31 and p_2 from the Redlich–Kwong equation 24.
(c) Calculate velocities C_{1n}, C_{1t}, and C_{2t} from Equations 8, 7, and 5, respectively, and C_{2n} from the continuity equation 3.
(d) Calculate the flow deflection angle δ from relation

$$\delta=\sigma-\arctan\frac{C_{2n}}{C_{2t}} \tag{36}$$

which results by combining Equations 9 and 10.
(e) Calculate C_2 from Equation 10.
(f) Calculate sound velocities α_1 and α_2 from Equation 28 and then Mach numbers $M_1=C_1/\alpha_1$ and $M_2=C_2/\alpha_2$.

Application for the real air

Application of the generalized (Redlich–Kwong based) method outlined above is made, as an example, in the case of real air. The data[15] required are the critical pressure $p_c=37.66$ bar and temperature $T_c=132.52$ K, the air constant $R=287.22$ J/kg K and the coefficients of the c_p' polynomial $A=1.0115846\times10^3$, $B=-1.0183346\times10^{-1}$, $C=2.7676571\times10^{-4}$, valid for temperatures 100–750 K. Extracts from the results obtained are shown in Figures 2 to 5, which correspond to stagnation pressure and temperature $p_{01}=10$ bar and $T_{01}=700$ K, respectively.

Figure 2 shows the calculated Mach number M_2, after the shock, in terms of the Mach number M_1, before the shock, with the angle σ as a parameter. Figure 3 shows the angle σ as a function of the deflection angle δ for various values of M_1. The ratios p_1/p_2, v_2/v_1 and T_1/T_2 in terms of M_1 for $\sigma=70$–$90°$ are given in Figure 4. Lastly, Figure 5 shows the calculated Rankine–Hugoniot relation for the shock wave of real air.

In all figures mentioned above, the results of the generalized

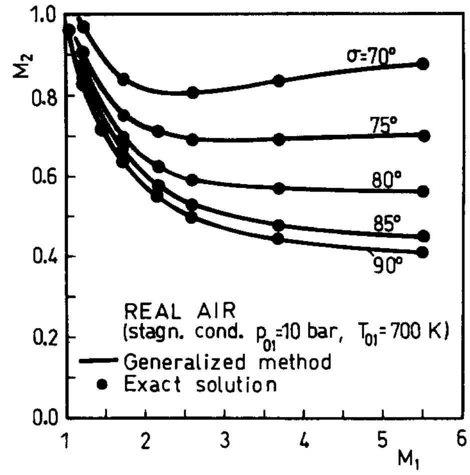

Figure 2 Calculated Mach number M_2, after the shock in terms of the Mach number M_1, before the shock, for various values of the angle σ

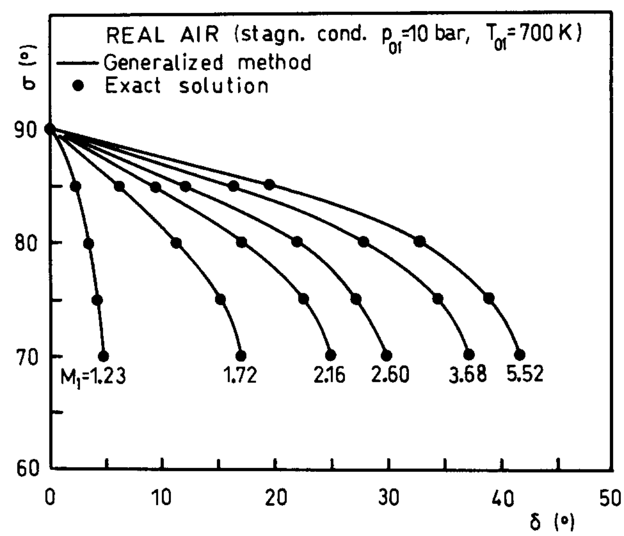

Figure 3 Angle σ in terms of the flow deflection angle δ, with Mach number M_1 as a parameter

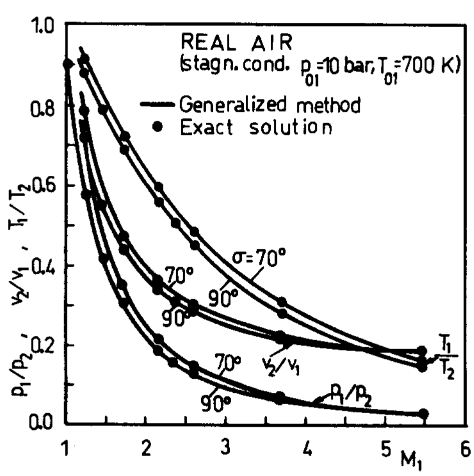

Figure 4 Calculated ratios p_1/p_2, v_2/v_1 and T_1/T_2 in terms of the Mach number M_1, for $\sigma=70°$–$90°$

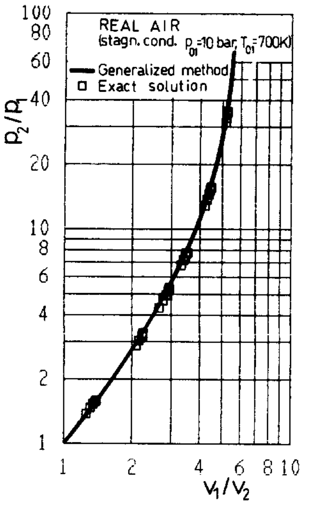

Figure 5 Calculated Rankine–Hugoniot relation

method developed are represented by solid lines. The point symbols correspond to the exact solution (see the next section) which is in excellent agreement with the generalized method.

Exact solution of the real gas oblique shock wave

Solution procedure

In this case the exact equation of state 1 and the enthalpy function 2 for the real gas considered should be available. Analytical expressions for the constant volume and constant pressure heat capacities c_v and c_p, and for the sound velocity α may be derived by using the exact equation of state $p=p(v, T)$ and relations[14,16]

$$c_v=c_p'-R+T\int_{\infty}^{v}\left(\frac{\partial^2 p}{\partial T^2}\right)_v \mathrm{d}v \tag{37}$$

$$c_p=c_v-T\left(\frac{\partial p}{\partial T}\right)_v^2 \bigg/ \left(\frac{\partial p}{\partial v}\right)_T \tag{38}$$

$$\alpha=\left[-v^2\frac{c_p}{c_v}\left(\frac{\partial p}{\partial v}\right)_T\right]^{0.5} \tag{39}$$

Substitution of p_2 and h_2 from the exact equation of state $p_2=p_2(v_2, T_2)$ and enthalpy function $h_2=h_2(v_2, T_2)$ into Equations 29 and 34, respectively, yields a set of two equations with two unknowns v_2 and T_2 (the angle σ is considered known, as in the previous method); i.e.,

$$f_1(v_2, T_2)=0 \tag{40}$$

$$f_2(v_2, T_2)=0 \tag{41}$$

The above set of equations is usually very complicated and may be solved only numerically. Various alternative algorithms may be devised for this purpose, or existing computer codes may be used. For example, the following steps (with appropriate refinements) may be followed:

(a) Take $v_2=v_1-\Delta v$, where Δv is a volume increment (i.e., $\Delta v=v_1/100$).
(b) Numerical solution of Equation 40 yields a value T_2' for the unknown temperature T_2.
(c) Numerical solution of Equation 41 yields a value T_2'' for the unknown T_2.
(d) If $T_2'=T_2''$, then the solution has been obtained. If $T_2'\neq T_2''$ decrease v_2 by Δv and repeat from step (b).

On the basis of the above analysis, the exact solution procedure for the real gas oblique shock wave may be summarized as follows:

(1) Give a value to σ ($0°<\sigma\leqslant 90°$) and calculate v_2 and T_2 by solving numerically the set of Equations 40 and 41 using the algorithm outlined above or any alternative one.
(2) Calculate C_{1n}, C_{1t}, C_{2t}, C_{2n}, δ, and C_2 from Equations 8, 7, 5, 3, 36, and 10, respectively.
(3) Calculate sound velocities α_1 and α_2 from Equation 39 and then Mach numbers $M_1=C_1/\alpha_1$ and $M_2=C_2/\alpha_2$.

Application for the real air

Application of the procedure outlined above is made in the case of real air for which the exact equation of state and enthalpy function are available;[15] i.e.,

$$p(v, T)=p_c\sum_{i=0}^{8}\left[A_{1i}\left(\frac{T}{T_c}\right)+A_{2i}+A_{3i}\left(\frac{T}{T_c}\right)^{-1}+A_{4i}\left(\frac{T}{T_c}\right)^{-2}\right]\left(\frac{v_c}{v}\right)^i \tag{42}$$

$$h(v, T)=pv+RT_c\sum_{i=0}^{9}\left[q_i\left(\frac{T}{T_c}\right)^i\right] + RT_c\sum_{i=0}^{7}\left[M_{1i}+M_{2i}\left(\frac{T}{T_c}\right)^{-1}+M_{3i}\left(\frac{T}{T_c}\right)^{-2}\right]\left(\frac{v_c}{v}-1\right)^i \tag{43}$$

where the values of the coefficients A, q, and M may be found in Ref. 15. Equations 42 and 43 are valid for $223\,\mathrm{K}\leqslant T\leqslant 1523\,\mathrm{K}$. Equations (37) and 38 take now the forms

$$c_v(v, T)=R\sum_{i=0}^{8}\left[B_i\left(\frac{T}{T_c}\right)^i\right] + R\sum_{i=0}^{7}\left[N_{1i}\left(\frac{T}{T_c}\right)^{-2}+N_{2i}\left(\frac{T}{T_c}\right)^{-3}\right]\left(\frac{v_c}{v}-1\right)^i \tag{44}$$

$$c_p(v\ T)=c_v+R\times\frac{\left[\sum_{i=0}^{7}[O_{1i}+O_{2i}(T/T_c)^{-2}+O_{3i}(T/T_c)^{-3}][(v_c/v)-1]^i\right]^2}{\sum_{i=0}^{7}[Q_{1i}+Q_{2i}(T/T_c)^{-1}+Q_{3i}(T/T_c)^{-2}+Q_{4i}(T/T_c)^{-3}][(v_c/v)-1]^i} \tag{45}$$

where the values of the coefficients B, N, O, and Q may be found in Ref. 15. Differentiation of Equation 42 with respect to v yields

$$\left(\frac{\partial p}{\partial v}\right)_T=p_c\times \sum_{i=0}^{8}\left[A_{1i}\left(\frac{T}{T_c}\right)+A_{2i}+A_{3i}\left(\frac{T}{T_c}\right)^{-1}+A_{4i}\left(\frac{T}{T_c}\right)^{-2}\right](-iv_c^i v^{-i-1}) \tag{46}$$

Substitution of c_v, c_p and $(\partial p/\partial v)_T$ from Equations 44, 45, and 46, respectively, into Equation 39 yields the sound velocity expression $\alpha=\alpha(v, T)$ for real air.

Following the exact solution procedure outlined earlier, the oblique shock wave of real air has been calculated under various conditions (stagnation pressure and temperature $p_{01}=10$–40 bar, $T_{01}=500$–1000 K, respectively). The computer time required was at least 20 times longer than that required for the generalized (Redlich–Kwong based) method. Extracts of the results (for stagnation conditions $p_{01}=10$ bar, $T_{01}=700$ K) are shown in Figures 2–5 (point symbols) together with the generalized solution (solid line), which is in excellent agreement. An additional test (not appearing in the figures) of the correctness of both solutions has been made by comparison with the exact solution of Ref. 1, which is available only for the limiting case of the normal shock wave ($\sigma=90°$, $\delta=0°$) of real air. In the cases examined, negligible differences of less than 0.1% appeared between the generalized and the exact solutions, while the two exact solutions did not differ at all, as expected.

Although experimental validation of the procedure was not easy owing to lack of suitable oblique shock wave data, the exact solution is considered to be very accurate since it is based on the experimentally derived exact equation of state and enthalpy function of real air.

Conclusion

The generalized (Redlich–Kwong based) method developed for calculating oblique shock waves of real gases seems to be very accurate, as deduced by the comparisons, shown in Figures 2 to 5, with the exact solution in the case of real air. The accuracy of the method is expected to be inferior in the region near the critical point, where the accuracy of the Redlich–Kwong equation diminishes (i.e., for reduced pressure and temperature $p_r = p/p_c = 1-1.1$, $T_r = T/T_c = 1-1.05$ the error of the Redlich–Kwong equation reaches 15%). The accuracy of the generalized method developed is limited only by the accuracy of the Redlich–Kwong equation of state. Although comparisons for other real gases (apart from air) have not been made, it is obvious that the present generalized method is accurate for those real gases whose behavior is well approximated by the Redlich–Kwong equation.

The calculations made for the real air showed also that the generalized method is at least 20 times faster than the exact solution. Therefore, although the method has been initially developed for real gases with insufficient thermodynamic information, it is recommended even if the exact equation of state and enthalpy function are available.

Worth noting are the derived real gas oblique shock wave equations, which retain the same algebraic form of the well-known ideal gas normal shock wave equations.

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